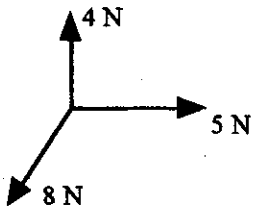
MECHANICS (A) UNIT 1 — TEST PAPER 4

Take $g = 9{\cdot}8\ \text{ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

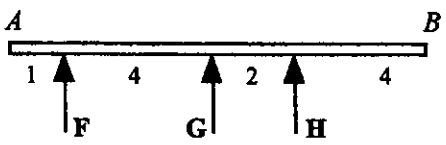
1. A tennis ball, moving horizontally, hits a wall at $25\ \text{ms}^{-1}$ and rebounds along the same straight line at $15\ \text{ms}^{-1}$. The impulse exerted by the wall on the ball has magnitude 12 Ns.
   (a) Calculate the mass of the ball. **(4 marks)**
   (b) State any modelling assumptions that you have made. **(2 marks)**

2. 

Forces of magnitude 4 N, 5 N and 8 N act on a particle in directions whose bearings are 000°, 090° and 210° respectively. Find the magnitude of the resultant force and the bearing of the direction in which it acts. **(7 marks)**

3. A packing-case, of mass 60 kg, is standing on the floor of a lift. The mass of the lift-cage is 200 kg. The lift-cage is raised and lowered by means of a cable attached to its roof.
   In each of the following cases, find the magnitude of the force exerted by the floor of the lift-cage on the packing-case and the tension in the cable supporting the lift :
   (a) The lift is descending with constant speed. **(3 marks)**
   (b) The lift is ascending and accelerating at $1{\cdot}2\ \text{ms}^{-2}$. **(4 marks)**
   (c) State any modelling assumptions you have made. **(2 marks)**

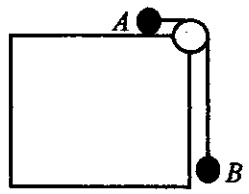
4. *AB* is a light rod. Forces **F**, **G** and **H**, of magnitudes 3 N, 2 N and 6 N respectively, act upwards at right angles to the rod in a vertical plane at points dividing *AB* in the ratio 1 : 4 : 2 : 4, as shown.



   A single force **P** is applied downwards at the point *C* to keep the rod horizontal in equilibrium.
   (a) State the magnitude of **P**. **(1 mark)**
   (b) Show that $AC : CB = 5 : 6$. **(5 marks)**

   Two particles, of weights 3 N and *k* N, are now placed on the rod at *A* and *B* respectively, while the same upward forces **F**, **G** and **H** act as before. It is found that a single downward force at the same point *C* as before keeps *AB* horizontal under gravity.
   (c) Find the value of *k*. **(6 marks)**

5. Two smooth spheres A and B, of masses $2m$ and m respectively, are connected by a light inextensible string which passes over a smooth fixed pulley as shown. A is initially at rest on the rough horizontal surface of a table, the coefficient of friction between A and the table being $\frac{2}{7}$. B hangs freely on the end of the vertical portion of the string. A is now given an impulse, directed **away from** the pulley, of magnitude $5m$ Ns.



(a) Show that the system starts to move with speed 2·5 ms^{-1}. **(1 mark)**

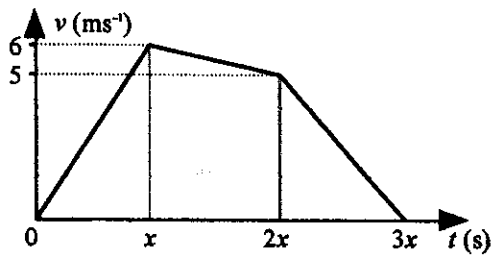
(b) State which modelling assumption ensures that the tensions in the two sections of the string can be taken to be equal. **(1 mark)**

Given that A comes to rest before it reaches the edge of the table and before B hits the pulley,

(c) find the time taken for the system to come to rest. **(7 marks)**

(d) Find the distance travelled by A before it first comes to rest. **(4 marks)**

6. The diagram shows the velocity-time graph for a cyclist's journey. Each section has constant acceleration or deceleration and the three sections are of equal duration x seconds each.



Given that the total distance travelled is 792 m,

(a) find the value of x and the acceleration for the first section of the journey. **(6 marks)**

Another cyclist covers the same journey in three sections of equal duration, accelerating at $\frac{1}{11}$ ms^{-2} for the first section, travelling at constant speed for the second section and decelerating at $\frac{1}{11}$ ms^{-2} for the third section.

(b) Find the time taken by this cyclist to complete the journey. **(6 marks)**

(c) Show that the maximum speeds of both cyclists are the same. **(2 marks)**

7. Relative to a fixed origin O, the points X and Y have position vectors $(4\mathbf{i} - 5\mathbf{j})$ m and $(12\mathbf{i} + \mathbf{j})$ m respectively, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) Find the distance XY. **(2 marks)**

A particle P of mass 2 kg moves from X to Y in 4 seconds, in a straight line at a constant speed.

(b) Show that the velocity vector of P is $(2\mathbf{i} + 1{\cdot}5\mathbf{j})$ ms^{-1}. **(3 marks)**

The particle continues beyond Y with the same constant velocity.

(c) Write down an expression for the position vector of P t seconds after leaving X. **(2 marks)**

(d) Find the value of t when P is at the point with position vector $(16\mathbf{i} + 4\mathbf{j})$ m. **(2 marks)**

When it is moving with the same constant speed, P collides directly with another particle Q, of mass 4 kg, which is at rest. P and Q coalesce and move together as a single particle.

(e) Find the velocity vector of the combined particle after the collision. **(5 marks)**

